

# Extended Rein–Sehgal model for tau lepton production

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The polarization density matrix formalism is employed to include the final lepton mass and spin into the popular model by Rein and Sehgal for single pion neutrino production. We investigate the effect of the  $\tau$  lepton mass on the differential cross sections. The lepton polarization evaluated within the extended RS model is compared against that follows from the single resonance production model based upon the Rarita-Schwinger formalism with phenomenological transition form factors.

## 1. INTRODUCTION

The Rein-Sehgal (RS) model [1,2] is undoubtedly one of the most circumstantial and approved phenomenological tools for description of single-pion production through baryon resonances in neutrino and antineutrino interactions with nucleons. It is incorporated into essentially all MC neutrino event generators in the few-GeV region, developed for both accelerator and astroparticle experiments [3]. However the RS model is not directly applicable to the  $\nu_\tau$  and  $\bar{\nu}_\tau$  induced reactions since it neglects the final lepton mass. Due to the same reason, the model is not suited for studying the lepton polarization phenomenon.

In Ref. [4], we proposed a generalization of the RS model which takes into account the final lepton mass and spin; it will be hereafter referred to as “Extended Rein-Sehgal” (ERS) model.

In this paper we briefly summarize the key points of the ERS model [4] (Sect. 2) and discuss some meaningful numerical results (Sect. 3).

## 2. SKETCH OF THE ERS MODEL

Our extension is based upon a covariant form of the charged leptonic current  $j_\lambda$  with definite lepton helicity  $\lambda$ , which allows us to express the components  $j_\lambda^\alpha$  of the current in the resonance rest frame (RRF) through the kinematic variables (and  $\lambda$ ) measured in

the laboratory frame. Since the leptonic current  $j_\lambda$  still can be treated as the intermediate  $W$  boson polarization 4-vector, it may be decomposed (in RRF) into three polarization 4-vectors  $e_{L,R,S}$  corresponding to left-handed, right-handed and scalar polarizations.

However, the vector  $e_S$  has to be modified with respect to that of the original RS model and, consequently, its inner products with the vector and axial charged hadronic currents  $F^{V,A}$  have to be recalculated. To do this, we used the explicit form for the currents  $F^{V,A}$  [5] of the Feynman-Kislinger-Ravndal (FKR) relativistic quark model [6] adopted in the RS approach. As a result, the three structures,  $S^V$ ,  $B^A$  and  $C^A$ , involved into the description of the FKR dynamics are also modified [4].

After that, the lepton polarization density matrix  $\rho = \|\rho_{\lambda\lambda'}\|$  can be written as the superposition of the partial cross sections  $\sigma_i^{\lambda\lambda'}$  ( $i = L, R, S$ ),

$$\rho_{\lambda\lambda'} = \frac{\Sigma_{\lambda\lambda'}}{\Sigma_{++} + \Sigma_{--}}, \quad \Sigma_{\lambda\lambda'} = \sum_{i=L,R,S} c_i^\lambda c_i^{\lambda'} \sigma_i^{\lambda\lambda'},$$

and the differential cross section is given by

$$\frac{d^2\sigma}{dQ^2 dW^2} = \frac{G_F^2 \cos^2 \theta_C Q^2}{2\pi^2 M |\mathbf{q}|^2} (\Sigma_{++} + \Sigma_{--}).$$

The cross sections  $\sigma_i^{\lambda\lambda'}$  are found to be bilinear combinations of the CC amplitudes referring to one single

resonance in a definite state of isospin, charge and helicity. These amplitudes remain *the same* as in the original RS model. The coefficients  $c_i^\lambda$  are explicitly defined through the components  $j_\lambda^\alpha$  written in RRF. The remaining kinematic variables and constants in the above equation have their standard meaning.

### 3. NUMERICAL RESULTS AND DISCUSSION

In our calculations, we use the same set of 18th nucleon resonances with masses below 2 GeV/ $c^2$  as in Ref. [1] but with all relevant parameters updated according to the most recent data [7]. The factors which were estimated in Ref. [1] numerically are corrected by using the new data and a more accurate integration algorithm. We do not modify the form of the transition form factors  $G^{V,A}(Q^2)$  adopted in the RS model but use the today's standard values for the axial mass and coupling constant.

In Fig. 1 we show, as an example, the finite lepton mass effect for the differential cross section  $d\sigma/dQ^2$  of the reaction  $\nu_\tau p \rightarrow \tau^- p \pi^+$ . The calculations are done for  $W < 2$  GeV. The major effect is, of course, due to the  $\tau$  lepton production threshold but the accounting for the mass in the lepton current (“dynamic correction”) gives rise to a significant additional decrease of the cross section: the effect can be as large as 300% at low neutrino energies and remains important up to rather high energies.

For a comparison, in Fig. 1 we also show the  $\nu_\mu p \rightarrow \mu^- p \pi^+$  cross section calculated in the ERS model. In this case, the dynamic mass correction is typically at the few per cent level or less and the purely kinematic correction for the muon mass is sufficient. The described situation is qualitatively similar for the rest  $\nu$  and  $\bar{\nu}$  induced reactions under consideration so we do not show the corresponding figures.

Figure 2 shows the double differential cross sections  $d^2\sigma/dP_\tau d\theta$  as functions of  $\tau$  lepton momentum  $P_\tau$  for six reactions of single pion production by  $\tau$  neutrinos and antineutrinos. Calculations are done in the ERS model. The choice for the incident  $\nu_\tau/\bar{\nu}_\tau$  energies  $E_\nu$  and the lepton scattering angles in the figure is quite arbitrary but representative. The distinctive shape of the cross sections with the two sharp Breit-Wigner cusps is a reflection of the two kinematically allowed solutions (“branches”) for  $P_\tau$  at fixed values of  $E_\nu$ ,  $\theta$  and  $W$ . So this feature is typical for

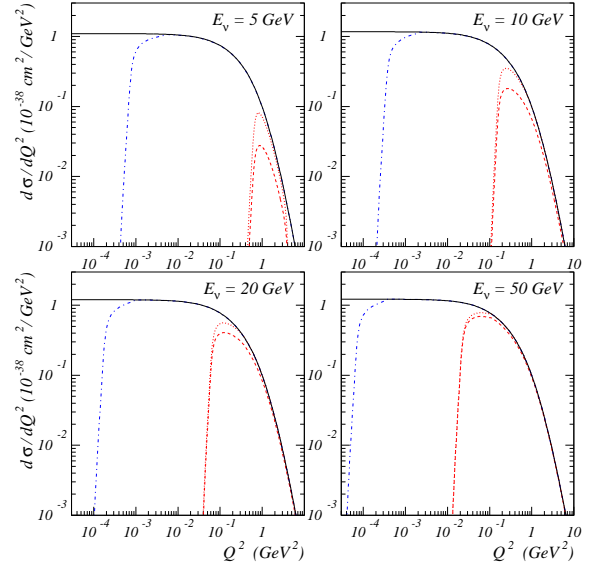


Figure 1. The finite lepton mass effect for the  $\nu_\tau p \rightarrow \tau^- p \pi^+$  differential cross section at  $E_\nu = 5, 10, 20$  and  $50$  GeV. Solid and dotted lines are, respectively, for the standard RS model predictions with zero lepton mass and with the  $\tau$  lepton mass included only into kinematics. Dashed lines are for the ERS model ( $m_\tau$  is taken into account in both kinematics and dynamics). The  $\nu_\mu p \rightarrow \mu^- p \pi^+$  cross section calculated in the ERS model is also plotted by dash-dotted lines. In all cases, the phase space has been cut by the condition  $W < 2$  GeV.

$d^2\sigma/dP_\tau d\theta$  at any allowed values of  $E_\nu$  and  $\theta$  and moreover it is much the same for the individual resonance contributions into the cross section. The amplitudes of the cusps and their ratio are however very responsive to variations of both  $E_\nu$  and  $\theta$ , e.g., near the reaction threshold, the contribution of the left cusp into the total cross section becomes important.

The fine structure of the cross sections is a result of the interference of many contributions with very different magnitudes and shapes. Since the resonance contributions are particular for each reaction, the fine structure is also quite distinctive.

For comparison, the cases for the single  $\Delta(1232)$  resonance production evaluated in the ERS model and

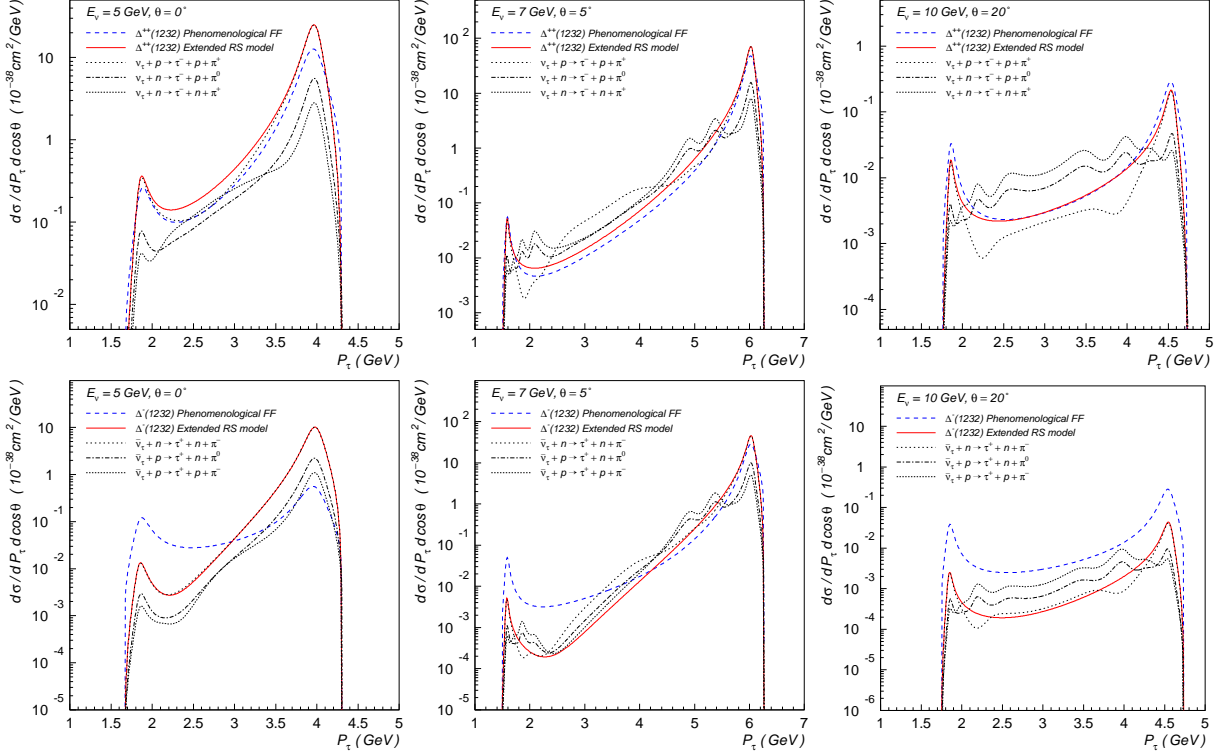


Figure 2. Double differential cross sections  $d^2\sigma/dP_\tau d\theta$ , as functions of  $\tau$  lepton momentum  $P_\tau$  for six reactions of single pion production by  $\nu_\tau$  (top panels) and  $\bar{\nu}_\tau$  (bottom panels). Calculations are done in the ERS model. The (anti)neutrino energies and scattering angles are indicated in the legends. The cases for the single  $\Delta(1232)$  resonance production evaluated in the ERS model and in the Rarita-Schwinger formalism with phenomenological transition form factors are also plotted.

in the Rarita-Schwinger formalism with phenomenological transition form factors as described in Ref. [4] are also shown. In the latter case we used the same input parameters (borrowed from Ref. [8]) as in the recent paper by Hagiwara et al. [9].

In Fig. 3 we plot the degree of polarization  $|\mathcal{P}|$  of  $\tau$  leptons generated in the same reaction and as in Fig. 2 at three scattering angles:  $\theta = 0^\circ, 5^\circ$  and  $10^\circ$ . As an example, the incident  $\nu_\tau/\bar{\nu}_\tau$  energy  $E_\nu$  is taken to be 7 GeV. It can be seen from the figure that, for the three  $\nu_\tau$  initiated reactions, the behavior of  $|\mathcal{P}|$  is qualitatively the same for any lepton momentum. For the three reactions initiated by  $\bar{\nu}_\tau$ , the shapes of  $|\mathcal{P}|$  are qualitatively comparable in the region of the main kinematic branch (the neighborhood of the right

cusps in Fig. 2) but become drastically different in the neighborhoods of the left cusps. The distinction of the individual profiles of  $|\mathcal{P}|$  for different reactions is again a result of the interference of different groups of the resonances with different amplitudes.

For comparison, the case with the single  $\Delta(1232)$  resonance production is also shown in Fig. 3. In this case, the degree of polarization was evaluated in the ERS model and by using the Rarita-Schwinger formalism with the phenomenological transition form factors suggested in Ref. [8]. As for the differential cross sections, we used the same set of inputs as in Ref. [9]. Our results are rather close to those of Ref. [9] but the agreement is not complete; in particular, Hagiwara et al. neglected the second kinematic

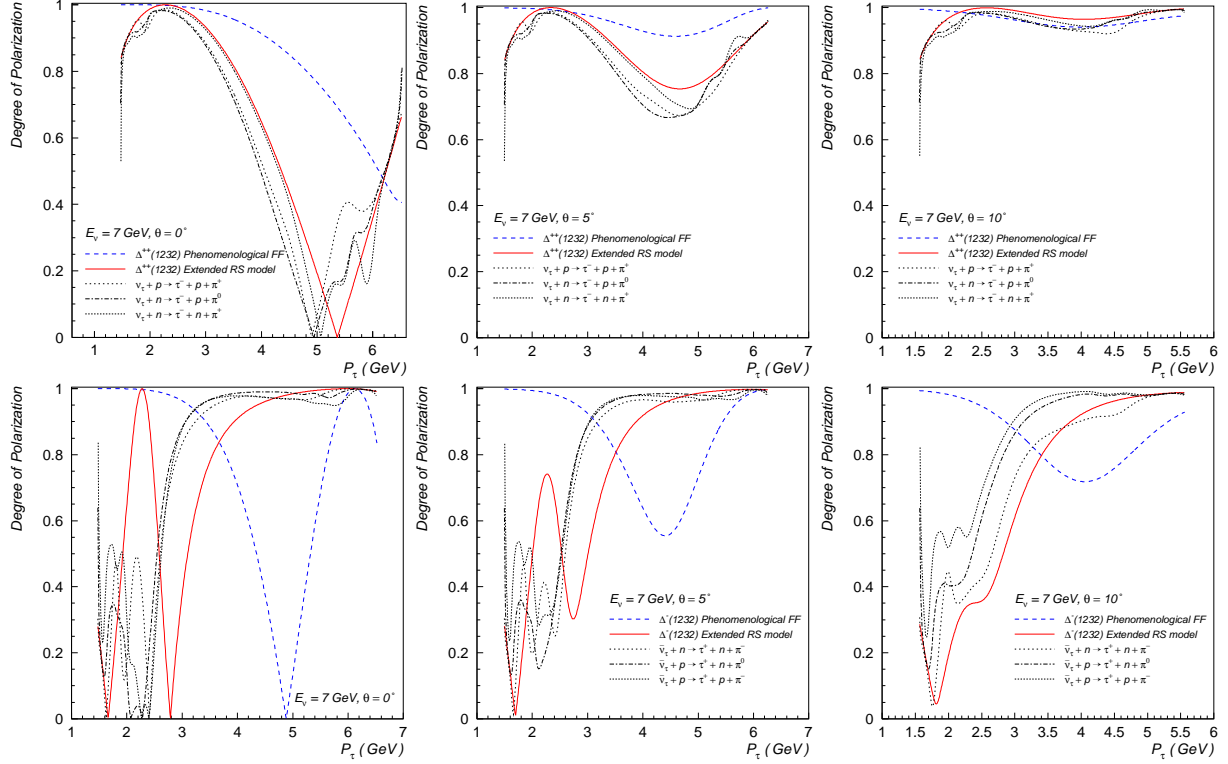


Figure 3. Degree of polarization of  $\tau$  leptons produced in the  $\nu_\tau$  and  $\bar{\nu}_\tau$  induced reactions at  $E_\nu = 7$  GeV for  $\theta = 0^\circ, 5^\circ$  and  $10^\circ$ . The cases for the single  $\Delta(1232)$  resonance production evaluated in the ERS model and in the Rarita-Schwinger formalism with phenomenological transition form factors are also plotted.

solution. It is however more important that the predictions of the ERS model and the Rarita-Schwinger approach for the single  $\Delta(1232)$  production are distinctly different for  $\nu_\tau$  initiated reactions (especially for small scattering angles and large lepton momenta) and completely different for  $\bar{\nu}_\tau$  initiated reactions (essentially everywhere).

Qualitatively similar conclusions can also be drawn for the longitudinal and perpendicular components of the polarization vector not illustrated in this paper.

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